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ON A CLASS OF UNBIASED ESTIMATORS USING MULTI-AUXILIARY INFORMATION

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SUMMARY

Following Srivastava [7] a class of unbiased estimators for the mean of a finite population using available multi-auxiliary information has been considered.

Keywords : Finite population; Auxiliary variables; Bias; Difference-cum-ratio estimator.

Introduction

Let Y_i, $X_{1i}, X_{2i}, \ldots, X_{pi}$ $(i = 1, 2, \ldots, N)$ denote values of the characters $y, x_1, x_2, \ldots, x_p$ respectively on the ith unit of a finite population of size N and

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i \text{ and } \bar{X}_k = \frac{1}{N} \sum_{i=1}^{N} X_{ki},\ k = 1, 2, \ldots, p.$$

The problem of estimating the population mean Y using p auxiliary variables $x_1, x_2, \ldots, x_p$ has attracted much attention. Using one auxiliary variable Srivastava [7] defined a class of estimators of $\bar{Y}$ considering a function of sample means of the study and auxiliary variable satisfying certain regularity conditions. But the class of estimators considered by Srivastava is quite restrictive since it does not cover the Hartley-Ross [2]-type estimators and those based on the mean of ratios or products etc.

In this paper, following Srivastava [7] a class of unbiased estimators for $\bar{Y}$, utilizing information on p auxiliary variables is defined. Suppose $\bar{X}_1$, $\bar{X}_2, \ldots, \bar{X}_p$ are known and a simple random sample $(y_i, x_{1i}, x_{2i}, \ldots, x_{pi})$ $(i = 1, 2, \ldots, n)$ of size n is drawn from the given population.

The Proposed Class of Estimators

Following Srivastava [7], let

$$g_i = h(y_i, x_{1i}, \ldots, x_{pi}) \tag{2.1}$$

be a function of $y_i, x_{1i}, x_{2i}, \ldots, x_{pi}$ $(i = 1, 2, \ldots, n)$ such that $h(\bar{Y}, \bar{X}_1, \ldots, \bar{X}_p) = \bar{Y}$. The function may contain $\bar{X}_k$ $(k = 1, 2, \ldots, p)$ but independent of $\bar{Y}$. Thus, a class of estimators for $\bar{Y}$ may be defined by

$$\hat{\bar{Y}}_g = \frac{1}{n} \sum_{i=1}^{n} g_i, \tag{2.2}$$

whose bias is given by

$$B = \frac{1}{N} \sum_{i=1}^{N} (g_i - y_i). \tag{2.3}$$

The estimation of bias involves in expressing B in a simpler form which further depends on the nature of the function $h(y_i, x_{1i}, \ldots, x_{pi})$. But sometimes the bias is in the form

$$B = \sum_{k=1}^{p} \theta_k \operatorname{Cov}(f_k, x_k),$$

where $f_1, f_2, \ldots, f_p$ are functions of $y, x_1, x_2, \ldots, x_p$ independent of $\bar{Y}$ and $\theta_1, \theta_2, \ldots, \theta_p$ are known constants.

Let $\hat{B} = \sum_{k=1}^{p} \theta_k \widehat{\operatorname{Cov}}(f_k, x_k)$ be an unbiased estimator of B. Then we consider the class of unbiased estimators of $\bar{Y}$ given by

$$t_U = \hat{\bar{Y}}_g - \hat{B}. \tag{2.4}$$

The class of estimators represented by t_U contains an infinite number of unbiased estimators. For example, the unbiased ratio estimator of Hartley and Ross [2], the unbiased product estimator based on the mean of the products considered by Gupta and Adhvaryu [1], the unbiased ratio-cum-product estimators of Sahoo and Swain [5], [6] and the multi-

variate unbiased ratio estimator of Olkin [4] are the members of the class. We also consider below some other interesting estimators of the class.
(i) Let us consider two auxiliary variables x_1 and x_2 and define

$$g = \frac{y - d(x_1 - \bar{X}_1)}{x_2} \bar{X}_2.$$

Then the bias of $\hat{\bar{Y}}_g$ is given by

$$-\operatorname{Cov}\left(\frac{y}{x_2}, x_2\right) - d\bar{X}_2 \operatorname{Cov}\left(\frac{1}{x_2}, x_1\right)$$

and $\hat{\bar{Y}}_g + \widehat{\operatorname{Cov}}\left(\frac{y}{x_2}, x_2\right) + d\bar{X}_2 \widehat{\operatorname{Cov}}\left(\frac{1}{x_2}, x_1\right)$ is an unbiased estimator for $\bar{Y}$. This is an unbiased difference-cum-ratio estimator. Similarly, one can construct an unbiased difference-cum-product estimator by taking

$$g = \frac{y - d(x_1 - \bar{x}_1)}{\bar{x}_2} x_2.$$

(ii) Let

$$g = y \frac{\sum_{k=1}^{p} w_k \bar{X}_k}{\sum_{k=1}^{p} w_k x_k}$$

Then, $t_U = \hat{\bar{Y}}_g + \sum_{k=1}^{p} w_k \widehat{\operatorname{Cov}}\left(\frac{y}{\sum_{k=1}^{p} w_k x_k}, x_k\right)$.

Similarly one may consider $g = y \frac{\sum_{k=1}^{p} w_k x_k}{\sum_{k=1}^{p} w_k \bar{X}_k}$ to get an unbiased product-type estimator, the type of which were studied by John [3].
(iii) Let

$$g = y \sum_{k=1}^{p} w_k \frac{x_k}{\bar{X}_k}.$$

Then $t_U = \hat{\bar{Y}}_g - \sum_{k=1}^{p} \frac{w_k}{\bar{X}_k} \widehat{\operatorname{Cov}}(y, x_k)$ is a p-variate unbiased product estimator for $\bar{Y}$.

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